Exam. Code : 103205
Subject Code : 1222

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—II

(Number Theory)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each section.

SECTION—A

I. (a) Prove that $2^{4n} - 1$ is divisible by 15. 2

(b) If x and y are positive integers and $x - y$ is even, show that $x^2 - y^2$ is divisible by 4. 4

(c) Show that square of an integer is of the form $3q$ or $3q + 1$ but not of the form $3q + 2$. 4

II. (a) Prove that $\gcd(a, b) = \gcd(a, b + ax) = \gcd(a + by, b)\ \forall\ x, y \in Z$. 5

(b) Find integers x & y so that $12378x + 3054y = 6$. 5

III. (a) Find general solution of the equation

$$91x + 221y = 1053.$$ 5

(b) Show that for each prime $p \geq 5$, $p^2 + 2$ is a composite number. 5

IV. (a) If $2^n + 1$ is an odd prime, show that n is equal to a power of 2. 5

(b) If $a \equiv b \pmod{m}$, then prove that $a + n \equiv b + n \pmod{m}$ and $an \equiv bn \pmod{m}$ for $n \in Z$. 5

V. (a) Show that $3^{287} - 3$ is divisible by 23. 5

(b) Solve $2x + 7y \equiv 5 \pmod{12}$. 5

SECTION—B

VI. (a) Solve $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$ by Chinese Remainder Theorem. 5

(b) Show that $a^7 \equiv a \pmod{42}$ for all $a \in Z$. 5

VII. (a) Prove that an integer $p > 1$ is a prime number iff $\lfloor p-2 \equiv 1 \pmod{p}$. 5

(b) Show that

$$\lfloor p-1 \equiv (p-1) \pmod{(1+2+3+\ldots+(p-1))}$$

for any prime p. 5

VIII. (a) For any odd prime p, show that $1^2.3^2.5^2. \ldots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}$ using Wilson's theorem. 5

(b) If p and $2p + 1$ both are odd primes and $m = 4p$, then prove that $\phi(m + 2) = \phi(m) + 2$. 5

IX. (a) Find positive integer m so that

$$\phi(2m) = \phi(m).$$ 5

(b) Find the last two digits in ordinary representation of 3^{400}. 5

X. (a) If x is real number, prove that

$$[x]+[-x]=\begin{cases} 0 & , \text{ if } x \in Z \\ -1 & , \text{ if } x \notin Z \end{cases}$$ 5

(b) Verify Mobius Inversion formula for n = 24. 5